APPLICATION OF GENERAL CUBIC B-SPLINE FUNCTION FOR GEOLOGICAL SURFACE SIMULATION

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ABSTRACT

Mathematical interpolation and smoothing are powerful tools which are being applied on many fields, especially for research on the Structural Geology with limited observation data. Because of its piecewise polynomial property, B-Spline function can avoid global dependences on local properties as well as be easy to derivate and integrate. General cubic B-Spline function with its advantages proves to be one of the effective choices for mathematic interpolation as well as smoothing. The intent of this research is to present an effective method of multivariate smooth fitting of scattered data using general cubic B-Spline function. An optimal smoothing function was defined to minimize the functional composed of data residuals and the first, second derivatives which represent the total misfit, fluctuation, and roughness of function, respectively. To define this optimal smoothing surface, penalty function method was used by changing parameter α. Theory of uniform cubic B-Spline functions with uniform knots was researched and generalized as General cubic B-Spline function (GCBSF) with free grid points. Because of this property, we can obtain the most information from limited observation data. A two dimension smoothing model using GCBSF was developed on Fortran 77 programming language. Some case studies were performed using given data to estimate optimal smoothing surfaces. The results show the applicability of this GCBSF model in simulation of geological surfaces.

1. INTRODUCTION

Geosciences field data are often noisy and measured at irregularly observational points because of practical problems. To have a general understand about continuous field of data, we must fit them with mathematical continuous functions. One of the approaches to this problem is to use smooth distribution property. Many methods for solving smoothing or interpolation problems have been proposed. Several of the mathematical method using piecewise continuous polynomials for trial functions have been carried out. Because of its piecewise polynomial property, B-Spline function can avoid global dependences on local properties as well as be easy to derivate and integrate. For some applications, the cubic B-Spline function (CBSF) with twice continuous derivative is acceptable. Therefore, CBSF with its advantages proves to be one of the effective choices for mathematical interpolation and smoothing. Inoue (1986) has proposed smooth fitting for irregularly spaced data using CBSF with equispaced knots. However, he has only mentioned about theory of uniform CBSF as well as tested for data on the equispaced grid. The aim of this research is to present a method of multivariate smooth fitting of catered data using general cubic B-Spline function for arbitrary grid and to develop a programming which can be easy to perform on personal computer with the given set of observational data.

2 SMOOTH FITTING USING GENERAL CUBIC B-SPLINE BASIS

Let $\Omega = \Omega_x \times \Omega_y$ be a rectangular domain in $x-y$ plane and $(x_p, y_p, z_p)(p=1,..,n)$ are n given observation data, where (x_p, y_p) are the defined points in Ω. Problem is to determine a smoothing function $f(x,y)$ defined in Ω and smooth fitted the data. $f(x,y)$ is a mathematical function and approximated by an expansion form with some trial function $F_i(x)$, $G_j(y)$ as

$$f(x,y) = \sum_{i=1}^{Mx}\sum_{j=1}^{My} c_{ij} F_i(x) G_j(y) \tag{2.1}$$

Firstly, suppose $J(f)$ be quantity which represents smoothness of the function and defined as

$$\begin{aligned} J(f) &= m_1 \iint_\Omega f_x(x,y)^2 + f_y(x,y)^2\, dxdy \\ &+ m_2 \iint_\Omega f_{xx}(x,y)^2 + 2f_{xy}(x,y)^2 + f_{yy}(x,y)^2\, dxdy \end{aligned} \tag{2.2}$$

where $f_x(x,y)$, $f_y(x,y)$, $f_{xx}(x,y)$, $f_{yy}(x,y)$, $f_{xy}(x,y)$ are first, second partial derivative and mixed partial derivative of $f(x,y)$ with respect to x and y, respectively, m_1 and m_2 are parameters.

Next, let $R_H(f)$ be misfit of the function $f(x,y)$ to the data in a form of residual mean of squares,

$$R_H(f) = \frac{\sum_{p=1}^{n}\left(f(x_p, y_p) - z_p\right)^2}{n} \tag{2.3}$$

The optimal smooth function $\overline{f}(x,y)$ can be defined to minimize the functional composed of the misfit and total roughness as

$$Q(f,\alpha) = J(f) + \alpha R_H(f) \tag{2.4}$$

where α is parameter that controls balance between the smoothness and the misfit of the function and α is called penalty. Substituting equation (2.1), (2.2) and (2.3) into (2.4) and taking $\frac{\partial Q}{\partial c_{ij}} = 0$ to minimize Q, we obtain a system of $Mx \times My$ linear equations for the coefficients c_{ij} as

$$Ac = \boldsymbol{b} \tag{2.5}$$

In this research, a general cubic B-Spline function was applied on the smooth fitting problem as trial function to approximate optimal smoothing function. Domain $\Omega = \Omega_x \times \Omega_y$ is divided into $\{x_1,..,x_{Mx}\}\times\{y_1,..,y_{My}\}$ and general cubic B-Spline bases $F_i(x)$ ($G_j(y)$) are written as

$$F_i(x) = \begin{cases} = 0 & x < x_{\mathrm{i}} \\ = B_1(x) = \left(\dfrac{x - x_i}{x_{i+3} - x_i}\right)\left(\dfrac{x - x_i}{x_{i+2} - x_i}\right)\left(\dfrac{x - x_i}{x_{i+1} - x_i}\right) & x_{\mathrm{i}} \le x < x_{\mathrm{i+1}} \\ = B_2(x) = \left(\dfrac{x - x_i}{x_{i+3} - x_i}\right)\left[\left(\dfrac{x - x_i}{x_{i+2} - x_i}\right)\left(\dfrac{x_{i+2} - x}{x_{i+2} - x_{i+1}}\right) + \left(\dfrac{x_{i+3} - x}{x_{i+3} - x_{i+1}}\right)\left(\dfrac{x - x_{i+1}}{x_{i+2} - x_{i+1}}\right)\right] + & \\ \quad \left(\dfrac{x_{i+4} - x}{x_{i+4} - x_{i+1}}\right)\left(\dfrac{x - x_{i+1}}{x_{i+3} - x_{i+1}}\right)\left(\dfrac{x - x_{i+1}}{x_{i+2} - x_{i+1}}\right) & x_{\mathrm{i+1}} \le x < x_{\mathrm{i+2}} \\ = B_3(x) = \left(\dfrac{x_{i+4} - x}{x_{i+4} - x_{i+1}}\right)\left[\left(\dfrac{x - x_{i+1}}{x_{i+3} - x_{i+1}}\right)\left(\dfrac{x_{i+3} - x}{x_{i+3} - x_{i+2}}\right) + \left(\dfrac{x_{i+4} - x}{x_{i+4} - x_{i+2}}\right)\left(\dfrac{x - x_{i+2}}{x_{i+3} - x_{i+2}}\right)\right] + & \\ \quad \left(\dfrac{x - x_i}{x_{i+3} - x_i}\right)\left(\dfrac{x_{i+3} - x}{x_{i+3} - x_{i+1}}\right)\left(\dfrac{x_{i+3} - x}{x_{i+3} - x_{i+2}}\right) & x_{\mathrm{i+2}} \le x < x_{\mathrm{i+3}} \\ = B_4(x) = \left(\dfrac{x_{i+4} - x}{x_{i+4} - x_{i+1}}\right)\left(\dfrac{x_{i+4} - x}{x_{i+4} - x_{i+2}}\right)\left(\dfrac{x_{i+4} - x}{x_{i+4} - x_{i+3}}\right) & x_{\mathrm{i+3}} \le x < x_{\mathrm{i+4}} \\ = 0 & x_{\mathrm{i+4}} \le x \end{cases} \tag{2.6}$$

where Mx and My are the number of the division in the domain. The gridding is arbitrary and depending on the distribution of data.

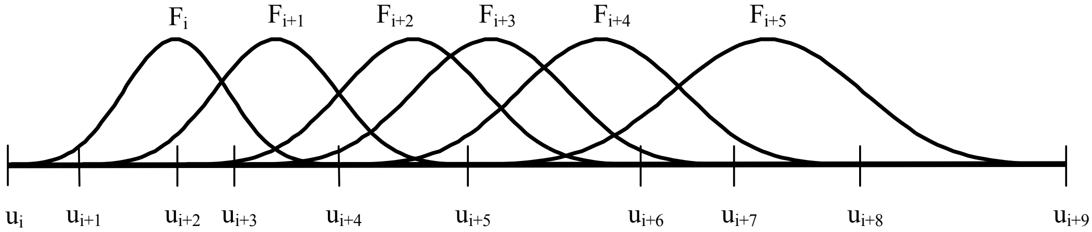

Figure 1. General cubic B-Spline bases with free knots

Thus, the right-hand coefficients of equation (2.5) can be calculated by

$$b_{ij} = \alpha \sum_{p=1}^{n} z_p F_i(x_p) G_j(y_p) \tag{2.7}$$

and the coefficient matrix $\boldsymbol{A}$ can be decomposed as

$$A = A^d + A^{10} + A^{01} + A^{20} + 2A^{11} + A^{02}$$

where: $$A^d_{iji'j'} = \alpha \sum_{p=1}^{n} F_i(x_p) G_j(y_p) F_{i'}(x_p) G_{j'}(y_p)$$

and $$A^{kl}_{iji'j'} = \int_{\Omega_x} F_i^{(k)}(x_p) F_{i'}^{(k)}(x_p)dx \int_{\Omega_y} G_j^{(l)}(y_p) G_{j'}^{(l)}(y_p)dy$$

The integration $A^{kl}_{iji'j'}$ is calculated analytically depending on each index $(i, j) \times (i', j')$ and not the same as in case of uniform cubic B-Spline. Because A is symmetric band matrix, equation (2.5) can be solved by elimination method using Cholesky factorization. The solution c_{ij} is used to build optimal smoothing surface which has formula as (2.1).

3 APLICATIONS

The model for smooth fitting using GCBSF is developed on Fortran 77 programming language. First the program is tested for fitting of surface to numerical example. Data were generated from function which has form as

$$f(x, y) = 1000 \times e^{-\frac{(x-45)^2 + (y-45)^2}{100}} \tag{3.1}$$

Figure 2 shows the surface fitted to the data points which are displayed by dots and the values. Next, a set of data modified from Table B.3 in Jones *et al.* (1986) is calculated, figure 3 shows the result which is surface fitted the data. All results are displayed as simulated surfaces by Terramod2001 software.

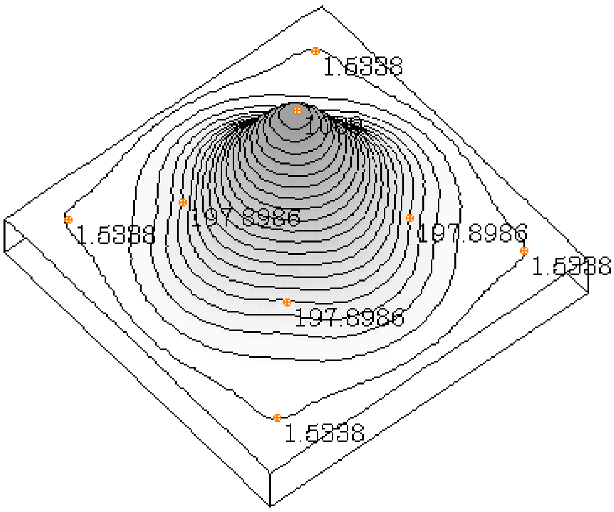

Figure 2. Surface fitted to nine data from test function

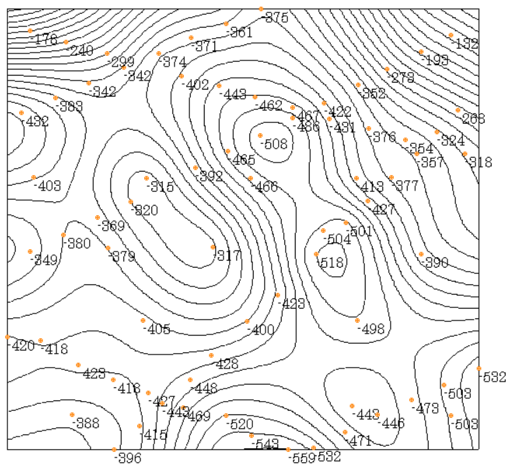

Figure 3. Surface fitted to data modified from Jones *et al.* (1986)

4 CONCLUSION AND FURTHER RESEARCH

The theory system of two dimension smooth fitting problem using General cubic B-Spline function is developed completely. The model based on this theory is built and coded in Fortran 77 language in order to determine smooth surfaces. The programming is tested using two given sets of data, the smooth simulated surfaces are showed in figures 2, 3. The results of calculation show the applicability of the GCBSF model in simulation of geological surfaces. However, using elimination method for solving the linear equation system (2.5) reaches the limit with some kinds of scattered data. In this research, the maximum number of equation is about 10000 equations. It means that there are about 100 grid points on each axis. For types of dense data, we will need other method which can solve a great many linear equations. Furthermore, this research needs more calculations for real data as well as other kinds of data that including more information about simulated objects. All above problems will be for future research.

5 REFERENCES

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